Math 101 Fall 2000 Final Exam

Instructor: Richard Stong

Wednesday, December 13, 2000

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **three hours**. Do all 12 problems. Please do all your work on the paper provided.
Please print you name clearly here.

Print name: ____

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

____

Grader's use only:

1. ________ /15

2. ________ /25

3. ________ /20

4. ________ /10

5. ________ /10

6. ________ /15

7. ________ /15

8. ________ /10

9. ________ /20

10. ________ /15

11. ________ /20

12. ________ /25

1. [15 points] Find the following limits, if they exist.

   (a) $\lim\limits_{x \to -1} \frac{x^2 - x - 2}{x^3 + 1}$

   (b) $\lim\limits_{x \to 0} \frac{1 - \cos 3x}{x \sin x}$

   (c) $\lim\limits_{x \to 0} (1 + 2x)^{3/x}$

2. [25 points] Find the derivatives of the following functions.

(a) $g(t) = t^2 \sec t$

(b) $k(t) = \frac{e^t}{\ln t}$

(c) $H(w) = \sqrt{\sin(2w)}$

(d) $h(x) = \arcsin(2x - 1)$

(e) $f(x) = \frac{(x-1)^{3/2}(x+2)}{(2x+1)\sqrt{x^2-1}}$

3. [20 points] Evaluate the following intergals.

(a) $\int (1 - 2\sqrt{x})^2 \; dx$

(b) $\int e^t \; \cos(3e^t) \, dt$

(c) $\int \frac{\tan(\sqrt{x})}{\sqrt{x}} \; dx$

(d) $\int \frac{x^2}{1+x^6} \; dx$

4. [10 points] Find the equation of the tangent line to the curve $y = \sqrt{4x+1}$ at the point $(2, 3)$.

5. [10 points] Solve the following initial value problem

$$\frac{dy}{dx} = 4\sin(2x); \;\; y(0) = 4.$$

6. [15 points] Find the area of the region in the plane bounded by the curves $y = x(x-3)^2$ and $y = x$. (Be careful, there are two pieces to this region.)

7. [15 points] Find the length of the curve $y = x^{3/2} - \frac{1}{3}x^{1/2}$ from $x = 1$ to $x = 4$.

8. [10 points] Let S be the surface that results from revolving the curve $x = \tan y$ from $y = 0$ to $y = \pi/3$ about the x-axis. Express the surface area of S as a definite integral, but do not attempt to evaluate the integral.

9. [20 points] Let R be the region in the plane bounded by the curve $y = \sqrt[3]{1+x}$, the line $x = 7$, and the line $y = 1$. Let S be the solid that results from revolving R about the x-axis. Express the volume of S as an integral in TWO ways, using the method of disks/washers and using the method of shells, and evaluate ONE of the two integrals (your choice).

10. [15 points] A bacteria culture contains 1000 bacteria 3 hours after it is started and 3000 bacteria 5 hours after it is started. Assuming exponential growth, find a formula for the number of bacteria present t hours after the culture was started and use this formula to estimate the initial number of bacteria present and the time when there will be 20,000 bacteria present.

11. [20 points] (a) We want to make a cylindrical can without a top which has a volume of 1000π in^3. What should the dimensions be in order to minimze the amount of material used to make the can (assuming no material is wasted in the construction of the can).

(b) Justify in a few lines why your answer to part (a) is really an absolute minimum.

12. [25 points] For the function $f(x) = \frac{x^2-3}{x+2}$, the first two derivatives are $f'(x) = \frac{x^2+4x+3}{(x+2)^2}$ and $f''(x) = \frac{2}{(x+2)^3}$. YOU ARE NOT REQUIRED TO VERIFY THESE FORMULAS. For all other aspects of this problem you are required to justify your answer.

(a) Find all horizontal and vertical asymptotes of the graph $y = f(x)$. Be sure to give the limits you need to show these are asymptotes.

(b) Find the open intervals on which the function f is increasing and those on which it is decreasing.

(c) Find all critical points of $f(x)$ and determine whether they are local maxima, local minima or neither. Justify your answer.

CONTINUED ON NEXT PAGE

(d) Find the intervals on which the function f is concave upward and those on which it is concave downward.

(e) Sketch the graph of $y = \frac{x^2-3}{x+2}$ using your answers to parts (a)-(d) and any additional information required.