Math 101 Fall 2001 Final Exam

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Thursday, December 13, 2001

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **three hours**. Do all 12 problems. Please do all your work on the paper provided.
Please print you name clearly here.

Print name: ______________________________

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

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Grader's use only:

1. ________ /15

2. ________ /30

3. ________ /20

4. ________ /30

5. ________ /20

6. ________ /10

7. ________ /10

8. ________ /15

9. ________ /15

10. ________ /10

11. ________ /10

12. ________ /15

1. [15 points] Evaluate the following limits, if they exist.

(a) $\lim_{x \to 3} \frac{x^2-4x+3}{x^2-x-6}$

(b) $\lim_{x \to 0} \frac{e^{x^2}-1}{1-\cos x}$

(c) $\lim_{x \to 1} x^{2/(x-1)}$

2. [30 points] Compute the derivatives of the following functions.

(a) $f(x) = x^3 \ln(x+3)$

(b) $g(t) = \sec(e^t)$

(c) $h(w) = \arctan(3w^2)$

(d) $G(x) = \int_1^{x^2} \frac{\sin t}{t} dt$

(e) $H(z) = (2 + (1 + 3\sin z)^3)^{1/2}$

3. [20 points] You want to build a rectangular box with no top, a square base and a volume of 500 cm^3. What dimensions will minimize the total surface area? Be sure to justify that your answer is really a global minimum.

4. [30 points] Let $f(x) = \frac{x^2-3}{x^2-1}$.

(a) Find all horizontal and vertical asymptotes of the graph $y = f(x)$. At the vertical asymptotes compute both the left and right hand limits of $f(x)$.

(b) Find the intervals on which $f(x)$ is increasing and those on which it is decreasing.

(c) Find the critical points of $f(x)$ and determine if they are local maxima or local minima.

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(d) Find the intervals on which $f(x)$ is concave upward and those on which it is concave downward.

(e) Sketch the graph of $y = \frac{x^2-3}{x^2-1}$ using your results in parts (a)-(d).

5. [20 points] Evaluate the following integrals.

(a) $\int (x^2 + 3)^2 dx$

(b) $\int_0^{\pi/2} \frac{\cos x}{2+\sin x} dx$

(c) $\int \sec^2 x \; e^{\tan x} dx$

(d) $\int \frac{x^3}{\sqrt{1-4x^8}} dx$

6. [10 points] Evaluate $\int_0^2 (1 + 3x^2)dx$ by computing $\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$. No credit will be given for computing the integral in any other manner.

7. [10 points] Find the area of the region in the plane bounded by $y = 4 - x^2$ and $y = x^2 - 2x - 8$.

8. [15 points] Suppose a particle on a line has velocity function $v(t) = t^2 - 4t + 3$ for $1 \leq t \leq 4$. Find the net distance travelled by the particle between $t = 1$ and $t = 4$ and the total distance travelled between $t = 1$ and $t = 4$.

9. [15 points] Let R be the region in the plane bounded by $x = 4y - y^2$ and the y-axis. Let S be the solid of revolution that results from revolving R about the y-axis. Express the volume of S as a definite integral in TWO ways, using the method of washers and the method of shells. Evaluate ONE of the two integrals (your choice).

10. [10 points] Find the length of the curve $y = 2(x-2)^{3/2}$ from $x = 2$ to $x = 9$.

11. [10 points] Express the area of the surface S obtained by revolving the curve $y = x^2$ for $0 \le x \le 3$ about the x-axis as a definite integral, but do not attempt to evaluate the integral.

12. [15 points] The population of Houston in 1960 was 1 million people and in 2000 it was 2 million people. Assuming exponential growth, find the population of Houston as a function of time. What will be the population of Houston in 2050? When will the population of Houston be 3 million?