

# Math 101 Fall 2002 Final Exam

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*Instructions:* This is a closed book, closed notes exam. Use of calculators is not permitted. You have **three hours**. Do all 12 problems. Please do all your work on the paper provided.

Please print your name clearly here.

Print name: \_\_\_\_\_

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam.

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Grader's use only:

1. \_\_\_\_\_ /25

2. \_\_\_\_\_ /15

3. \_\_\_\_\_ /20

4. \_\_\_\_\_ /10

5. \_\_\_\_\_ /10

6. \_\_\_\_\_ /15

7. \_\_\_\_\_ /15

8. \_\_\_\_\_ /20

9. \_\_\_\_\_ /15

10. \_\_\_\_\_ /20

11. \_\_\_\_\_ /20

12. \_\_\_\_\_ /15

1. [25 points] Evaluate the derivatives of the following functions

(a)  $g(x) = \frac{\sin x}{1 + \tan x}$

(b)  $h(t) = \sqrt{1 + e^{-2t}}$

(c)  $F(x) = \arcsin(1 - x^2)$

(d)  $G(t) = (1 + \sec(t^3 - 1))^2$

(e)  $H(x) = \int_2^{x^2} \frac{1}{\ln t} dt$

2. [15 points] Evaluate the following limits, if they exist.

(a)  $\lim_{x \rightarrow -2} \frac{x^3 + x^2 + 4}{x^2 + x - 2}$

(b)  $\lim_{x \rightarrow 0} (e^{2x} - 1) \cot x$

(c)  $\lim_{x \rightarrow \infty} \left(1 - \frac{2}{x}\right)^{x+2}$

3. [20 points] Evaluate the following integrals

(a)  $\int 3xe^{x^2+1} dx$

(b)  $\int_0^\pi \frac{\sin x}{3-2\cos x} dx$

(c)  $\int \frac{x^2}{1+x^6} dx$

(d)  $\int (1 + \tan x)^3 \sec^2 x dx$

4. [10 points] Find the area of the region in the plane bounded by the curves  $y = x - x^2$  and  $y = 2x - 2$ .

5. [10 points] Find the equation of the tangent line to the function  $y = \ln(1 + 2x)$  at  $x = 0$ .

6. [15 points] Let  $R$  be the region in the plane bounded by the curve  $y = \sqrt{x} - 2$ , the line  $x = 9$  and the  $x$ -axis. Let  $S$  be the solid that results from revolving  $R$  about the  $x$ -axis. Express the volume of  $S$  as a definite integral in TWO ways, using the method of washers and the method of shells. Evaluate ONE of these two integrals (your choice).

7. [15 points] Consider the curve  $C$  given by  $y = (e^x + e^{-x})/2$  for  $0 \leq x \leq 2$ .

(a) Find the length of the curve  $C$ .

(b) Express the area of the surface that results from revolving  $C$  about the  $y$ -axis as a definite integral, but do not attempt to evaluate the integral.

8. [20 points] For the function  $f(x) = \frac{1}{5}x^5 + \frac{1}{4}x^4 - \frac{8}{3}x^3 - 6x^2$ , the first two derivatives are  $f'(x) = x^4 + x^3 - 8x^2 - 12x = x(x+2)^2(x-3)$  and  $f''(x) = 4x^3 + 3x^2 - 16x - 12 = (4x+3)(x+2)(x-2)$ . YOU ARE NOT REQUIRED TO VERIFY THESE FORMULAS. For all other aspects of this problem you are required to justify your answer.

(a) Find the open intervals on which the function  $f$  is increasing and those on which it is decreasing.

(b) Find all critical points of  $f(x)$  and classify them as local maxima, local minima or neither.

(c) Find the intervals on which the function  $f$  is concave upward and concave downward.

9. [15 points] When salt is dissolved in water, the amount  $A$  of salt that remains undissolved after  $t$  minutes satisfies the differential equation  $\frac{dA}{dt} = -kA$ , ( $k > 0$ ). If 25% dissolves after 2 minutes, how long does it take for half the salt to dissolve? NOTE: you do not have to simplify your final answer.

10. [20 points] We wish to make an ice cream cone with a volume of  $18\pi$  in<sup>3</sup>. What dimensions give the minimum possible area? You must justify that your answer really is the global minimum. (The area of a cone of height  $h$  and radius  $r$  is  $A = \pi r\sqrt{r^2 + h^2}$  and the volume is  $V = \frac{1}{3}\pi r^2 h$ .)

11. [20 points] The volume of a cone is  $\frac{1}{3}\pi r^2 h$  where  $r$  is the radius and  $h$  is the height. Suppose the radius and height of a cone are changing with time. In particular the radius is increasing at a constant rate of 2 cm/s while the height is decreasing at a constant rate of 3 cm/s. When the radius is 4cm and the height is 6cm at what rate is the volume changing?

12. [15 points] Compute the definite integral  $\int_0^2 (1+3x^2)dx$  from the definition, that is, by finding  $\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i)\Delta x$ . No credit will be given for computing the integral in any other way.