Math 101 Fall 2000 Exam 1

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Tuesday, October 3, 2000

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all 8 problems. Please do all your work on the paper provided.
Please print you name clearly here.

Print name: _____

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

_____

Grader's use only:

1. _____ /15

2. _____ /10

3. _____ /10

4. _____ /10

5. _____ /20

6. _____ /15

7. _____ /10

8. _____ /10

1. [15 points] Find the following limits, if they exist.

   (a) $\lim_{x \to -2} \frac{x^2 - x - 6}{x + 2}$

   (b) $\lim_{x \to 0} \frac{\tan 5x}{x}$

2. [10 points] Let f be the function defined by

$$f(x) = \begin{cases} 3 - x & \text{if } x < 2 \\ 0 & \text{if } x = 2 \\ 2x^2 - 7 & \text{if } x > 2 \end{cases}$$

Find $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2^-} f(x)$, and $\lim_{x \to 2} f(x)$ (if they exist). Is f continuous at $x = 2$?

3. [10 points] (a) Give the formal, mathematical definition of the derivative of a function f.

   (b) Find the derivative of $f(x) = \frac{1}{x+2}$ **using the definition of the derivative**. (No credit will be given for finding the derivative by other means.)

4. [10 points] Find the equation of the tangent line to the graph of $y = \sqrt[3]{x}$ at $x = 8$.

5. [20 points] Find the derivatives of the following functions.

(a) $f(x) = 1 + 3\sqrt{x} + 2x^2 - 6x^{-3}$

(b) $g(t) = \frac{e^{2t+1}}{1+3t^2}$

(c) $F(t) = \sqrt{t}\sin(t^4)$

(d) $f(x) = \left(2\ln(2+3x^{-2}) + 7\right)^8$

6. [15 points] A sector is removed from a circular piece of cardboard of radius 10 cm. The remaining cardboard is folded so the ends of the sector join to form a cone. What is the maximum possible volume of the resulting cone? (The volume of a cone with height h and radius of the base r, is $V = \frac{1}{3}\pi r^2 h$.)

7. [10 points] Use implicit differentiation to find $\frac{dy}{dx}$ if $x \tan y = \cos(x + y)$.

8. [10 points] A baseball diamond is a square with side length 90 ft. A batter hits the ball and runs towards first base with a velocity of 24 ft/sec. At what rate is his distance from 3rd base increasing when he is halfway to first base?