

Math 101 Fall 2001 Exam 1

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Tuesday, September 25, 2001

Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all 7 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.

Please print your name clearly here.

Print name: _____

Upon finishing please sign the pledge below:

On my honor I have neither given nor received any aid on this exam.

Grader's use only:

1. _____ /15

2. _____ /10

3. _____ /10

4. _____ /20

5. _____ /10

6. _____ /20

7. _____ /15

1. [15 points] Find the following limits, if they exist.

(a) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 3x + 2}$

(b) $\lim_{\theta \rightarrow 0} \frac{\sin 2\theta}{\tan 3\theta}$

(c) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

2. [10 points] Let f be the function defined by

$$f(x) = \begin{cases} \frac{3-x}{|x-3|} & \text{if } x < 3 \\ -1 & \text{if } x = 3 \\ x^2 - 4x + 4 & \text{if } x > 3 \end{cases}$$

Find $\lim_{x \rightarrow 3^+} f(x)$, $\lim_{x \rightarrow 3^-} f(x)$, and $\lim_{x \rightarrow 3} f(x)$ (if they exist). Is f continuous at $x = 3$?

3. [10 points] Find the derivative of $f(x) = \frac{1}{3x-1}$ **using the definition of the derivative**. (No credit will be given for finding the derivative by other means.)

4. [20 points] Find the derivatives of the following functions.

(a) $f(x) = \frac{3x-7}{x^2+4x+1}$

(b) $g(t) = (1 + t^2)^3 \cdot \sqrt{2 + t}$

(c) $y(x)$ if $y = \frac{1}{2u^2} + u^2$ and $u = x^{1/3} - \frac{1}{3x}$

(d) $F(x) = (3 - (2 + 5x^2)^6)^{-1/5}$

5. [10 points] Find the equation of the tangent line to the graph of $y = 3x - \frac{1}{x}$ at the point $(1, 2)$.

6. [20 points] Consider the function $f(x) = x \left(\frac{5}{2} - \frac{x}{2}\right)^{2/3}$ on $[1, 7]$. Find the maximum and minimum of f on this interval. Be sure to show all the steps you need to show in order to justify that your answers really are the maximum and minimum. (It may help to know that $f(1) = 2^{2/3} = 1.5874\dots$)

7. [15 points] A poster is to have an area of 600 in^2 with 1 inch margins at the bottom and sides and a 2 inch margin on top. What dimensions will give the largest printed area?