Math 101 Fall 2001 Exam 2

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Instructions: This is a closed book, closed notes exam. Use of calculators is not permitted. You have **one hour and fifteen minutes**. Do all 8 problems. Please do all your work on the paper provided. You must show your work to receive full credit on a problem. An answer with no supporting work will receive no credit.
Please print you name clearly here.

Print name: ______________________________

Upon finishing please sign the pledge below:
On my honor I have neither given nor received any aid on this exam.

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Grader's use only:

1. ________ /15

2. ________ /20

3. ________ /10

4. ________ /10

5. ________ /15

6. ________ /10

7. ________ /5

8. ________ /15

1. [15 points] Find the following limits, if they exist.

(a) $\lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x^2}$

(b) $\lim_{x \to \pi} \cos(x/2) \cot(x)$

(c) $\lim_{x \to 0} (1 - 3x)^{1/(2x)}$

2. [20 points] Find the derivatives of the following functions.

(a) $f(x) = x^2 e^{\sin 2x}$

(b) $g(t) = \sec(7t^2 + 1)$

(c) $F(x) = \arcsin(\ln x)$

(d) $G(x) = x^{\sqrt{x}}$

3. [10 points] Find the first three derivatives of the following function:

$$f(x) = \cos(2\sqrt{x})$$

4. [10 points] The function $y(x)$ is defined (implicitly) by the equation

$$\sin(x + 2xy) = x^2 + y^2.$$

Find $\frac{dy}{dx}$.

5. [15 points] A kite is flying at an altitude of 80 ft and is carried horizontally by the wind at a rate of 5 ft/sec. At what rate is string released to maintain this flight when 100 ft of string has been released?

6. [10 points] A particle moves along the x-axis with acceleration function $a(t) = \sin(t/2)$, initial position $x(0) = 3$, and initial velocity $v(0) = 0$. Find the particle's position $x(t)$ as a function of time.

7. [5 points] Express $\sum_{i=1}^{n} (2i-1)^2$ as a polynomial in n.

8. [15 points] For the function $f(x) = \frac{x^2+2x+5}{x+1}$, the first two derivatives are $f'(x) = \frac{(x+3)(x-1)}{(x+1)^2}$ and $f''(x) = \frac{8}{(x+1)^3}$. YOU ARE NOT REQUIRED TO VERIFY THESE FORMULAS. For all other aspects of this problem you are required to justify your answer.

(a) Find the intervals on which the function f is increasing and those on which it is decreasing.

(b) Find all critical points of $f(x)$ and determine whether they are local maxima, local minima or neither. Justify your answer.

(c) Find the intervals on which the function f is concave upward and those on which it is concave downward.